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MAGNETOHYDRODYNAMIC PRESSURE DROP IN DUCTED TWO-PHASE FLOWS

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Abstract—Two models are presented for predicting magnetohydrodynamic pressure drop in two phase gas–liquid flows of conducting fluids for large values of Hartmann number. The first of these models treats the gas–liquid mixture as a single homogeneous pseudofluid with averaged mixture properties. The second model assumes that the flow pattern is one where the liquid is displaced to the duct walls as a liquid film and the gas flows in the central core. It is shown that the two models do not differ significantly in their predictions of overall pressure drop for vaporising two-phase flow of potassium. There is little experimental data available for testing the models but very satisfactory agreement is found between measurements of magnetic pressure drop of NaK–nitrogen mixtures at low quality and the predictions of both models.

1. INTRODUCTION

During the last 5 years, a great deal of effort has been expended both in analysing magneto-hydrodynamic single phase duct flows and in experimentally verifying the results of these analyses (Hunt & Shercliff 1971; Branover & Tsinsober 1970). Very little attention has, however, been directed to the prediction of gas–liquid two-phase magneto-hydrodynamics. This is not surprising—the highly empirical nature of two-phase flow analyses gives little hope for the prediction of two phase MHD flows without extensive experimental data, which is not currently available. One limiting case is, however, amenable to study; that of two-phase flow in which the Hartmann number for the liquid phase is very large (electro-magnetic stresses dominate the viscous stresses).

This limiting case, moreover, has practical significance. For example, in a steady state deuterium–tritium fusion reactor, a high temperature reacting plasma is magnetically confined and produces the majority of its reaction energy as 14 MeV neutrons. These neutrons are thermalised in a lithium bearing blanket from which heat has to be recovered at a sufficiently high temperature for the efficient generation of electrical power. One proposed method for cooling this lithium blanket is to employ potassium as a heat-transfer fluid. Liquid potassium is pumped through the toroidal confining magnetic field into the lithium blanket region in which it is boiled and from which it emerges as a vapour which is passed through a turbine or heat exchanger. The computation of pressure losses in such a primary coolant circuit requires knowledge of the pressure drop occurring in the MHD two-phase boiling region.

In this paper, two simple models are presented for the computation of pressure drop in MHD two-phase flow at high Hartmann numbers. A comparison is made between the predictions of the two models for the case of potassium boiling at constant heat flux in a uniform duct. The predictions of each model are also compared with some limited experimental data for pressure drop in the two-phase MHD flow of NaK–nitrogen mixtures in a vertical rectangular duct.

2. FLOW MODELS

2.1 *Homogeneous model*

This model assumes that the two-phase flow may be adequately represented by a single-phase flow having pseudo-properties calculated by suitably weighting the properties of the individual phases. An equivalent two-phase electrical conductivity has been derived by Hall–Taylor (1967) as:

$$\sigma_{TP} = \sigma_L/(1 + 1300X) \qquad [1]$$

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based on experimental data for air–water and potassium. In [1], σ_{TP} is the equivalent electrical conductivity of the two-phase mixture, σ_L is the electrical conductivity of the liquid phase and X is the mass quality of the flow. This form of correlation† implies that the two-phase electrical conductivity is independent of the density ratio of the phases. Strictly, this cannot be true, but though there is probably a weak dependence of σ_{TP} on the density ratio ρ_L/ρ_G, use of [1] should be reasonably accurate for most systems. An equivalent two-phase volumetric flowrate is defined:

$$Q_{TP} = Q\left[1 + X\left\{\frac{V_G - V_L}{V_L}\right\}\right] \qquad [2]$$

where Q_{TP} is the two-phase volumetric flowrate, Q is the equivalent liquid volumetric flowrate, V_G is specific volume of the gas phase and V_L is the liquid specific volume. If σ_W is the electrical conductivity of the duct wall, t is the wall thickness and r the hydraulic radius of the duct, an equivalent wall conductance ratio for two-phase flow ϕ_{TP}, may be defined:

$$\phi_{TP} = \left[\frac{\sigma_w t}{\sigma_{TP} r}\right] = \phi\,(1 + 1300X), \qquad [3]$$

provided there is no contact resistance between the fluid and the wall. In [3], ϕ is the wall conductance ratio for a pure liquid flow. An equivalent two-phase Hartmann number M_{TP}, is given by:

$$M_{TP} = rB\,(\sigma_{TP}/\nu_{TP})^{1/2} \qquad [4]$$

where B is the magnetic field intensity transverse to the flow and ν_{TP} is th equivalent two-phase kinematic viscosity. If ν_{TP} is equated with the liquid phase kinematic viscosity ν_L, the equivalent two-phase Hartmann number becomes:

$$M_{TP} = M_L/(1 + 1300X)^{1/2} \qquad [5]$$

where M_L is the liquid phase Hartmann number.

The homogeneous model for two-phase pressure drop at high Hartmann numbers yields (Hunt & Hancox 1971):

$$\left.\frac{\partial p}{\partial z}\right|_{TP} = \frac{-Q_{TP}\sigma_{TP}B^2}{A}\left\{\frac{\phi_{TP}}{1 + \phi_{TP}}\right\}. \qquad [6]$$

In [6] $\partial p/\partial z|_{TP}$ is the two-phase axial pressure gradient and A is the duct cross-sectional area. For mass qualities of $0.05 < X < 1$, $\phi_{TP} \gg 1$ then:

$$\left.\frac{\partial p}{\partial z}\right|_{TP} = \frac{Q_{TP}\sigma_{TP}B^2}{A} = \left.\frac{\partial p}{\partial z}\right|_{L} \frac{\left[1 + X\left\{\frac{V_G - V_L}{V_L}\right\}\right]}{1 + 1300\,X}. \qquad [7]$$

Equation [7] directly relates the two-phase pressure gradient to the pressure gradient which would exist if liquid only flowed through the duct at the same mass velocity as the two-phase mixture. The latter is readily computed from well-established equations (Hunt & Hancox 1971).

A more sophisticated approach is to specify a "flow pattern" within the framework of an idealised representation. It, therefore, appears necessary to direct attention to a flow pattern in which there is a liquid continuum adjacent to the duct wall while in the centre of the duct there is a

†Alternative forms of correlation express the two-phase electrical conductivity as a function of void fraction (Fujiie & Suita 1974).

vapour continuum. Such a flow is commonly termed "annular flow" (Hewitt & Hall-Taylor 1970) and typically occurs over a substantial portion of the boiling region in a duct. In this paper, the terms "annular flow" and "film flow" will denote a flow in which the liquid is displaced to the heated walls and the vapour is a continuum in the central core of the duct.

2.2 *Two-phase "annular" MHD flow*

Consider a rectangular duct with an applied transverse uniform magnetic field (figure 1). A conducting fluid of electrical conductivity σ flows through the duct and is initally all liquid. To the top and bottom faces of the duct is applied a constant heat flux h. Attention is focussed on the

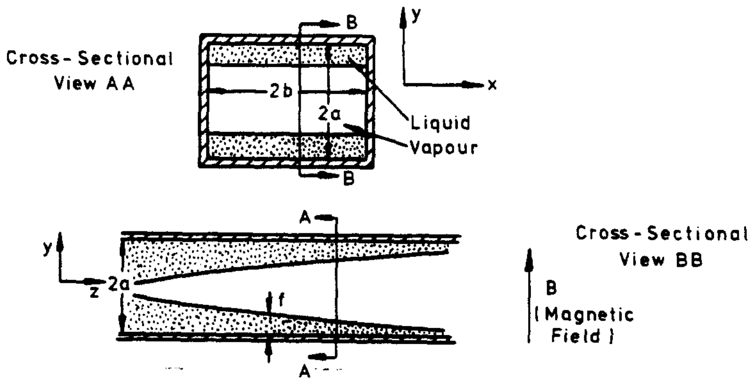

Figure 1. Assumed flow pattern for the film flow model.

two-phase region. The first bubbles of vapour formed at the duct surface will tend to produce a bubbly flow but an "annular" flow regime should be quickly established. The flow is characterised by gradual thinning of the liquid film and increasing vapour flow—the liquid phase experiences a large MHD force opposing its motion while the vapour flow exerts a large interfacial drag force on the liquid film. To make analysis tractable, the "annular" flow condition will be assumed to commence at the inception of vapour formation. For large values of Hartmann number (i.e. provided $fB(\sigma/\nu)^{1/2} > 130\, fU_L/\nu$) the large magnetic damping forces in the liquid phase will prevent the onset of turbulence in the liquid film. In the above inequality, f is the liquid film thickness, and U_L is the liquid phase velocity. Wave formation at the vapour–liquid interface may be suppressed, greatly decreasing phase interaction and entrainment. The simplifying assumption is made that the vapour liquid interface is smooth and that there is no entrainment from the liquid film to the vapour core.

The effect of the magnetic field on the two-phase flow may now be considered. The motion of the conducting fluid induces an electric field $\mathbf{U} \times \mathbf{B}$ in the liquid phase, where $\mathbf{U}$ is the velocity and $\mathbf{B}$ the local magnetic flux density. Typical current paths for two-phase "annular" flow in a rectangular duct are shown in figure 2. If the walls of the duct are electrical insulators the currents generated in the bulk of the liquid film return through the film itself. If the duct walls are highly conducting then the currents return through the duct walls. The electric currents induce their own magnetic field which is mainly directed along the duct because the currents flow in transverse loops. The ratio of this induced field to the imposed magnetic field is known as the Magnetic Reynolds number and is usually small.

Consider the flow at a cross section such as A–A in figure 1. Conservation of mass in the duct requires:

$$\rho_G \int_0^{(a-f)} U_G \, dy + \rho_L \int_{(a-f)}^{a} U_L \, dy = \frac{\dot{m}}{4b} \qquad [8]$$

where ρ_G is the gas-phase density, ρ_L is the liquid-phase density, a is the half width of the duct, b

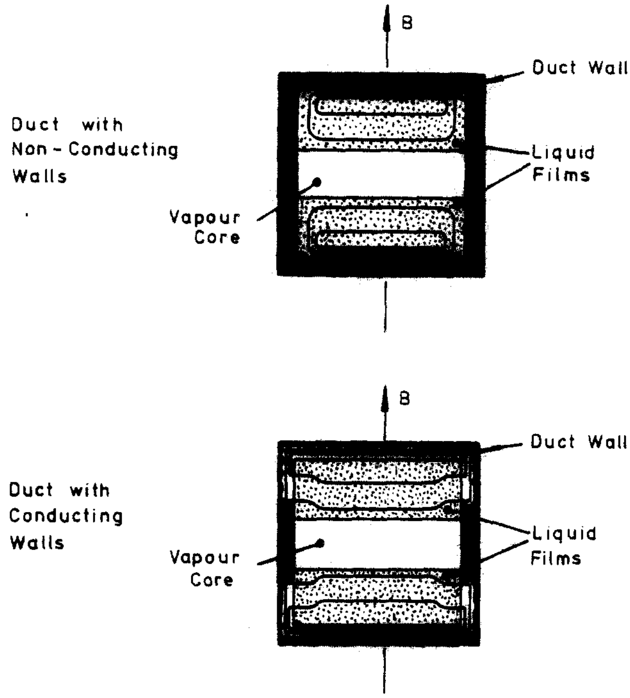

Figure 2. Electric current paths in two-phase flow due to a strong transverse magnetic field. The velocity is out of the page.

is the half breadth of the duct and $\dot{m}$ is the total mass flow through the duct. The liquid-phase velocity is U_L. If the fluid properties and saturation temperature are only slowly varying along the channel, an elemental heat balance yields:

$$h = \lambda \rho_L \frac{\mathrm{d}}{\mathrm{d}z}\left\{\int_{(a-f)}^{a} U_L \,\mathrm{d}y\right\} \qquad [9]$$

which reduces to:

$$h = -\lambda \rho_L \left\{ U_L|_{y=(a-f)} \frac{\mathrm{d}f}{\mathrm{d}z} + \int_{(a-f)}^{a} \frac{\partial U_L}{\partial z}\,\mathrm{d}z \right\}, \qquad [10]$$

where λ is the latent heat of vaporization per unit mass, y is the transverse space coordinate and z is the axial coordinate. If the liquid flow is assumed to be laminar, an elemental force balance for the film may be written as:

$$-\frac{\partial p}{\partial z} + \sigma E_x B - \sigma U_L B^2 + \mu_L \frac{\partial^2 U_L}{\partial y^2} = 0, \qquad [11]$$

where E_x is the induced electric field. For the gas phase, provided acceleration effects are not significant, a force balance exists between the pressure gradient and the shear force at the vapour–liquid interface:

$$-\frac{\partial p}{\partial z} = \frac{C \rho_G \bar{U}_G^2}{(a-f)}, \qquad [12]$$

where C is a frictional coefficient, evaluated assuming that the vapour flows through an essentially stationary duct. For the large gas to liquid relative velocities which would be characteristic of these MHD flows, this approximation should be accurate. The frictional coefficient is estimated from the results for smooth-walled tubes (the liquid surface may have waves on it which would increase the drag coefficient):

$$C = 16/Re_G; \quad 0 < Re_G \leqslant 2000, \tag{13a}$$

$$C = \frac{1}{4}\{0.86859 \ln [Re_G/1.964 \ln Re_G] - 3.8215\}^{-2} \quad \text{for} \quad Re_G > 2000. \tag{13b}$$

In [13], Re_G is the gas phase Reynolds number, defined $4\bar{U}_G(a-f)b/\nu_G$. Induced eddy currents are assumed to circulate in the x–y plane (this is only true provided U_L and f vary slowly along the duct) thus, if the duct walls are thin, of thickness t and conducting with electrical conductivity σ_w:

$$\sigma \int_{(a-f)}^{a} (E_x - U_L B)\,\mathrm{d}y = -E_x\sigma_w t, \tag{14}$$

but:

$$\frac{\partial E_x}{\partial y} = 0. \tag{15}$$

Hence:

$$E_x = B \int_{(a-f)}^{a} U_L\,\mathrm{d}y / f(1+\phi_f), \tag{16}$$

where ϕ_f is the relative conductance of the duct wall to the film $(\sigma_w t/\sigma_L f)$. From [16] it may be observed that $(\partial E_x/\partial z) \neq 0$ and since $(\partial E_z/\partial x) = (\partial E_x/\partial z)$, a current j_z is induced in the film. This implies that the pressure within the duct varies with the x-coordinate so that the thickness of the liquid film varies across the duct. This effect is small, however, because change in E_x takes place over a distance large compared with "a" or "b".

Equations [8], [9], [11], [12] and [16] comprise a set of five equations for the parameters U_L, $\bar{U}_G$, f, E_x, $(\partial p/\partial z)$.

The following boundary conditions apply. Liquid film velocity is zero at the solid surfaces hence:

$$y = \pm a; \qquad U_L = 0. \tag{17a}$$

There is no discontinuity in the velocity profile at the gas–vapour interface (a discontinuity would imply a locally infinite shear stress);

$$y = \pm(a-f); \qquad U_L = U_G. \tag{17b}$$

The shear stress exerted by the liquid on the vapour phase at the interface, τ_I, must equal the shear stress exerted by the vapour flow on the liquid film:

$$\rho_L\nu_L \left.\frac{\partial U_L}{\partial y}\right|_I = \rho_G\nu_G \left.\frac{\partial U_G}{\partial y}\right|_I = \tau_I. \tag{17c}$$

The film flow of liquid at any section must equal the total liquid flow at inlet less the quantity vaporised. Thus:

$$U_L f = -\int_0^z \frac{h}{\lambda\rho_L}\,\mathrm{d}z + aU_L\bigg|_{z=0}. \tag{17d}$$

2.3 *Solution procedure*

For large liquid film Hartmann numbers, viscous effects in the film may be neglected so that the velocity in the film U_L will be independent of y. In fact, as illustrated in figure 3, there are thin Hartmann layers both on the wall and on the liquid–air interface. These Hartmann layers have a characteristic thickness of (f/M) where f is the liquid film thickness and M is the Hartmann number.

Combining [11] and [16] yields:

$$-\frac{\partial p}{\partial z} - \sigma U_L B^2 \left\{1 + \frac{1}{1 + \phi_f}\right\} = 0. \qquad [18]$$

Similarly, [8] and [12] may be combined to give:

$$U_L = \left\{\frac{\dot{m}}{4b} - \frac{\rho_G{}^{1/2}(a - f)^{3/2}\left(-\frac{\partial p}{\partial z}\right)^{1/2}}{C^{1/2}}\right\} \Big/ \rho_L f. \qquad [19]$$

Equations [18], [17] and [19] are solved iteratively using a Newton–Raphson procedure for specified values of $\dot{m}$, h, λ, a, b, B, σ_w, t, σ, ρ_G and ρ_L.

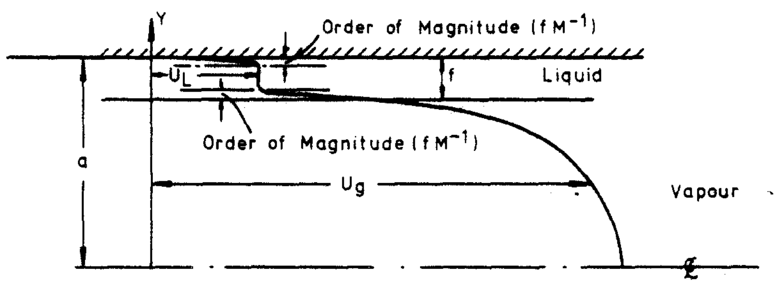

Figure 3. The velocity profiles for the gas and liquid phases.

3. DISCUSSION OF RESULTS

Sample results are presented for the boiling of potassium in a retangular duct.

Figures 4, 5 and 6 present the variation of U_L, f, E_x, $(\partial p/\partial z)$ and $\dot{m}_G$ with axial position for values of input parameters given below in table 1. It may be observed that the film flow model predicts that the liquid film remains appreciably thick for a considerable region and then rapidly thins. When the liquid film becomes thin, viscous and turbulent effects dominate and the flow becomes like the usual viscous dominated two-phase annular flow.

For thick films, MHD film flow behaves differently from the more common viscous film flow. In the latter case the film is reduced both by evaporation and by axially increasing interfacial shear forces whereas in the former case film thickness is determined by a local balance between MHD holdup and evaporation effects.

Table 1. Input parameters for two-phase boiling potassium

Parameter	Numerical value	Parameter	Numerical value
$\dot{m}$	0.00127 (kg/s)	t	0.1×10^{-2} (m)
h	2×10^4 (W/m^2)	ρ_G	0.53 (kg/m^3)
λ	1.87×10^6 (J/kg)	ρ_L	662.2 (kg/m^3)
a	1×10^{-2} (m)	μ_G	2.49×10^{-5} (kg/ms)
b	2×10^{-2} (m)	σ	1.35×10^6 (Ω^{-1}/m)
B	5.0 (T)	$(V_G - V_L)/V_L$	1300 (—)
σ_w	5×10^6 (Ω^{-1}/m)	μ_L	1.29×10^{-4} (kg/ms)

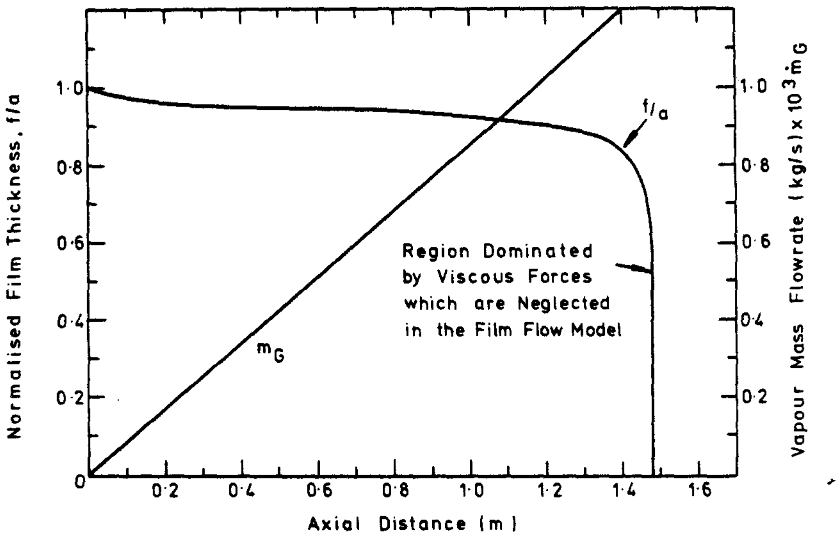

Figure 4. Liquid film thickness and vapour flowrate variation with axial distance as computed by the film flow model.

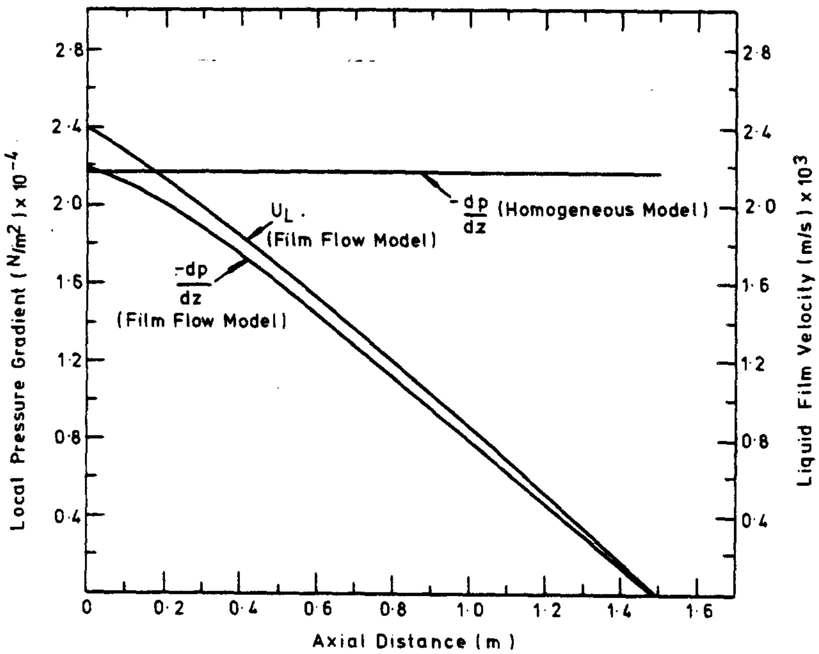

Figure 5. Pressure gradient and liquid velocith variation in the two-phase boiling region.

Figures 4, 5 and 6 show that the MHD film model predicts that the pressure gradient decreases in a near linear manner to the single phase value at the point of dryout. At a constant applied heating rate, the mass flow of vapour increases linearly. The slightly anomolous behaviour of the film flow model at very low qualities is attributable to the laminar–turbulent transition in the gas phase—in reality the flow pattern would be bubbly at such low qualities.

Comparisons have been made between the pressure drops predicted by the two models for boiling flow of potassium in a rectangular duct. Graphs 7–10 show the effect of various parameters on predicted pressure-drop in a two-phase boiling potassium flow. From these figures, it may be observed that for a given duct;

(1) The discrepancy between predicted pressure gradients remains approx constant as the heat flux increases.

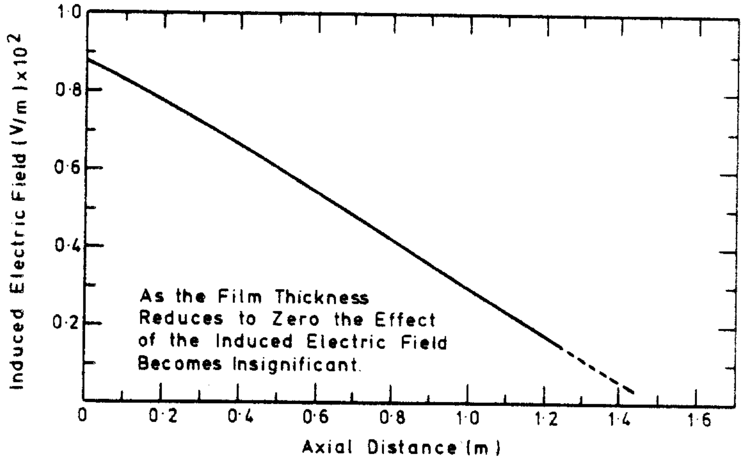

Figure 6. Induced electric field variation with axial distance.

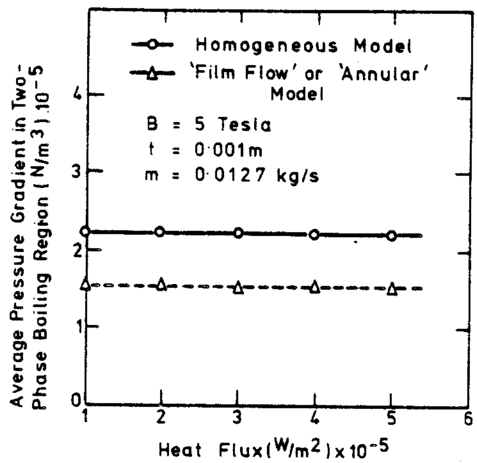

Figure 7. The effect of heat flux on predicted average pressure gradient in the two-phase boiling region.

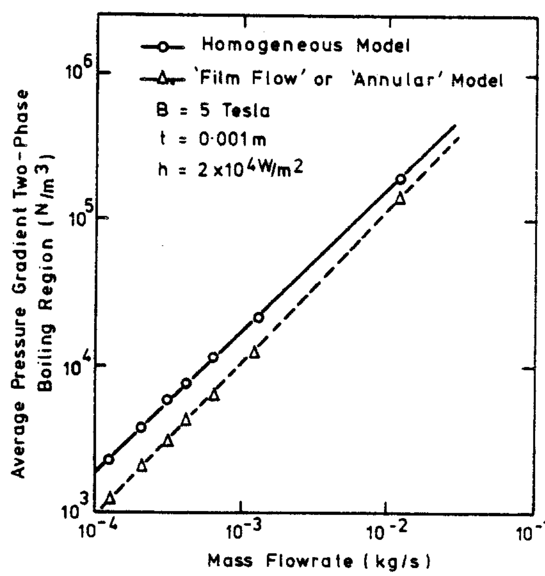

Figure 8. The effect of potassium mass flowrate on predicted average pressure gradient in the two-phase region.

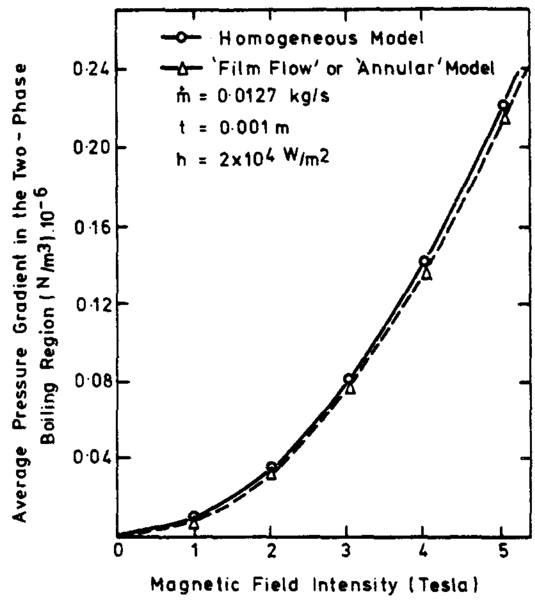

Figure 9. The effect of magnetic field intensity on the average pressure gradient in the two-phase boiling region.

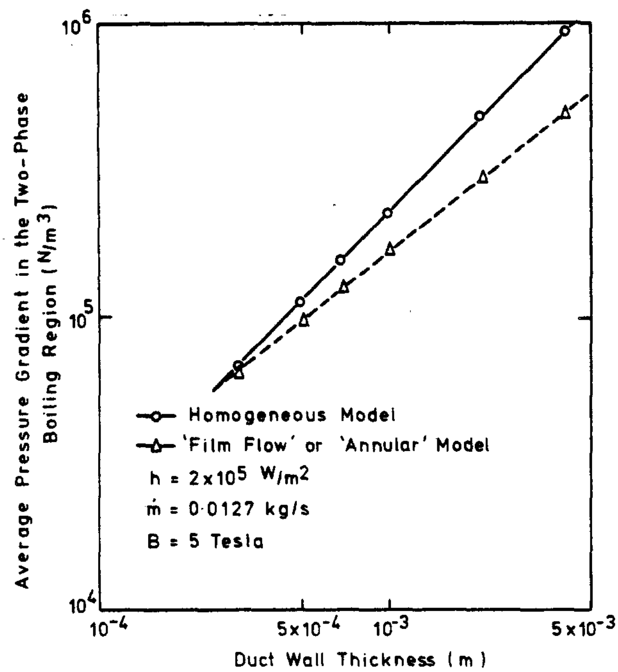

Figure 10. The effect of wall thickness on the average pressure gradient in the two phase boiling region.

(2) As the mass flow is decreased, the discrepancy between the predicted pressure gradients tends to increase.

(3) The disagreement in predicted pressure gradients between the homogeneous model and the film model is, the first approximation, independent of magnetic field intensity.

(4) The discrepancy between predicted pressure gradients tends to increase as the wall thickness increases.

These observations indicate the effect of different parameters on the pressure gradient predicted by two different models of two-phase flow in a duct at high Hartmann numbers. The model in which a specific flow pattern is assumed (the film flow pattern, in this case) generally predicts lower pressure gradients in boiling potassium than does the model in which the phases are considered as a homogeneous mixture.

There is little experimental data available with which to test the above models. To the authors' knowledge the only reported experimental data is that of Thome (1964) and this data is limited to very small mass qualities. Thome measured the magnetic pressure drop for two-phase flow of NaK (78% K, 22% Na) and nitrogen in a vertical rectangular duct, (0.052 m, 0.0064 m). A magnetic field of maximum intensity 0.784 tesla was applied to a 0.102 m length of the duct. The proposed analytical models have been compared with the data of Thome for an NaK flowrate of 0.378 kg/s and a magnetic field strength of 0.784 tesla (figure 12). For the homogeneous model, the single phase liquid MHD pressure drop was computed from:

$$\Delta p = 0.1032\, \sigma_L B^2 U_L \left\{\frac{1}{M_L} + \frac{\phi_L}{1+\phi_L}\right\} + \sigma_L B^2 U_L b (d')^{1/2} \qquad [20]$$

where d' is the wall conductivity ratio based on b.

In figure 11, the above formula is shown to agree closely with Thome's single phase experimental data. The second term in [20] is an "end-effect" correction. A similar end-effect correction was applied to the pressure drops predicted by the film flow model and allows for the region of decreasing magnetic field strength outside the magnetic poles. It may be observed that both analytical models predict pressure drops which compare very satisfactorily with the values derived from direct measurements. It should be noted, however, that the flowrates in Thome's experiments are such that $fB(\sigma/\nu)^{1/2} < 130 fU_L/\nu$ so that the liquid film is probably turbulent.

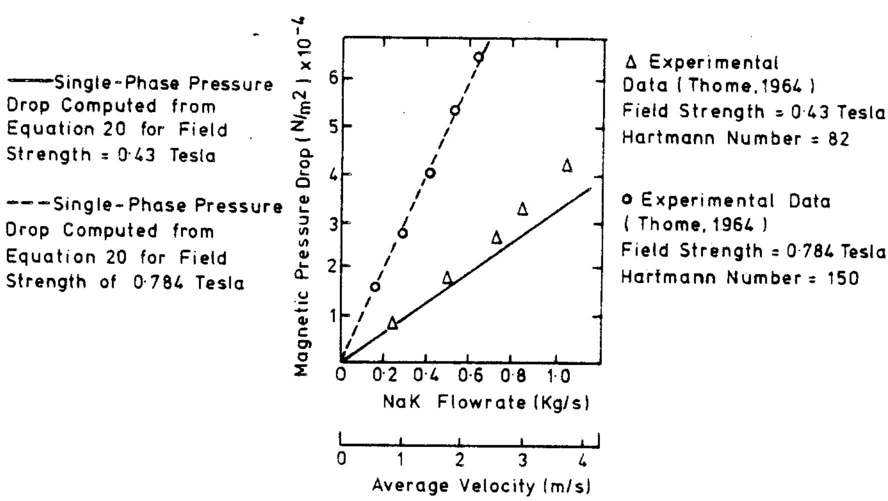

Figure 11. Magnetic pressure drop in single-phase flow of NaK through a rectangular duct.

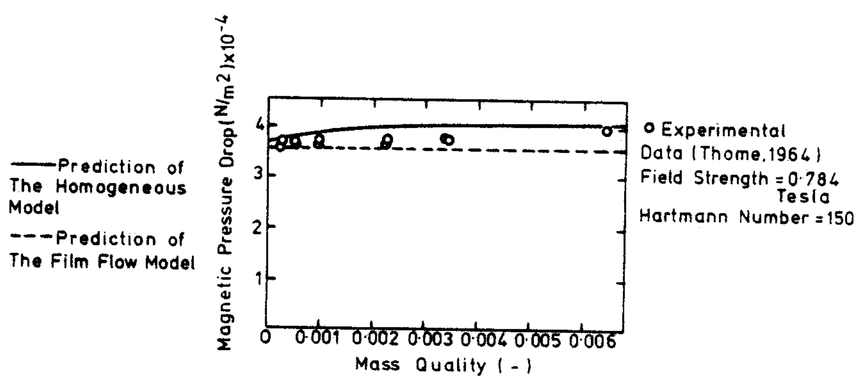

Figure 12. A comparison of the homogeneous and film flow models with experimental data for NaK and nitrogen mixtures flowing in a rectangular duct.

CONCLUSIONS

A simple homogeneous model of two-phase MHD pressure drop at large Hartmann numbers is presented and compared with the predictions of a more sophisticated film flow model. For once-through, induct vaporisation of potassium the two models usually predict overall two-phase pressure gradients which agree within 50% although, in certain cases, discrepancies up to 100% are observed.

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